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Biggest mystery in mathematics in limbo after cryptic meeting

Confusion still surrounds *abc* conjecture, but Oxford gathering boosts prospects for resolution.

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A collective effort to scrutinize one of the biggest mysteries in mathematics has ended with a few clues but no firm answers.

AP/Press Association Image

Mathematician Shinichi Mochizuki made a Skype appearance at a workshop about his work.

The mystery concerns an impenetrable but potentially groundbreaking proof — of a puzzle known as the *abc* conjecture — that appeared online three years ago. Whether the proof is valid is still not clear — a source of frustration for some of the leading specialists who gathered at the University of Oxford on 7–11 December to discuss the matter.

Others say that the workshop, in which the proof's reclusive architect Shinichi Mochizuki made a rare, virtual appearance, has at least boosted prospects for a resolution.

The quest to understand Mochizuki's proof dates back to August 2012, when he quietly posted four papers on his website in which he claimed to have solved the *abc* conjecture. The problem gets its name from expressions of the form $a + b = c$ and connects the prime numbers that are factors of a and b with those that are factors of c. Its solution could potentially change the face of number theory, which deals with the fundamental properties of, and relationships between, whole numbers.

Cryptic tome

But Mochizuki's papers, which totalled more than 500 pages[1–4], were exceedingly abstract and cryptic even by the standards of pure mathematics. That has made it tough for others to read the proof, let alone verify it. Moreover, the papers built on an equally massive body of work that he had accumulated over the years, but that few were familiar with.

Mochizuki, who is 46 and a highly respected member of the Research Institute for Mathematical Sciences (RIMS) at Kyoto University in Japan, does not like to travel and has rejected all invitations to lecture about his papers outside of Japan. So far, only a handful of researchers have managed to read his proof, and most had to spend long periods with Mochizuki in Kyoto. It is an overwhelming task even for the leading mathematicians in Mochizuki's branch of number theory, known as arithmetic geometry.

The biggest mystery in mathematics: Shinichi Mochizuki and the impenetrable proof

The workshop aimed to reboot the process of scrutiny. It covered both Mochizuki's preliminary work and an outline of his four *abc* papers. The contents of the papers were presented in large part by two researchers who say that they have checked the proof in its entirety — number theorists Yuichiro Hoshi and Go Yamashita, both from RIMS. True to form, Mochizuki himself did not attend, although he did answer participants' questions through Skype. The workshop was hosted by the Clay Mathematics Institute, a non-profit organization housed in the University of Oxford's main mathematics building.

A consensus emerged that the highlight of the workshop was a lecture on 9 December by Kiran Kedlaya, an arithmetic geometer from the University of California, San Diego. He zeroed in on a result from a 2008 paper by Mochizuki[5] that linked the statement of the *abc* conjecture to another branch of maths called topology. The link was immediately recognised as a crucial step in Mochizuki's grand strategy.

Aha! moment

Seeing this was the type of 'Aha!' moment that researchers were waiting for, says number theorist Brian Conrad of Stanford University

in California, but the rest of the conference failed to build on this success.

Even Kedlaya agrees that the insight needs to be followed up by many others, and by an understanding of the strategy that links those key passages to one another. “There is still no clear answer to lingering questions about how things are ultimately going to fit together,” he says. Still, he says, he now feels motivated to invest more time into vetting Mochizuki’s proof, and hopes to help streamline it in the process.

Most of the workshop attendees had been mystified about Mochizuki’s proof before the workshop, says Minhyong Kim, an arithmetic geometer at the University of Oxford and an organizer of the meeting. Now, at least, some have an idea of the general strategy and they have narrowed down the objects of their confusion to specific parts of the proof, he says.

But Conrad and many other participants say they found the later lectures indigestible. Kim counters that part of the difficulty lay in cultural differences: Japanese mathematicians have a more formal style of lecturing than do those in the West and they are not as used to being questioned by a testy audience, he says.

Paradox at heart of maths makes physics problem unanswerable

Mathematical theatre

Others complained that not all of the content of the early lectures, which examined Mochizuki's preliminary work, was actually necessary for what came later on at the workshop. “The decision about which topics to cover lacked some overall understanding of the proof,” says Jakob Stix, a number theorist at the Goethe University in Frankfurt, Germany. “Which is not really a complaint, because I sense that nobody really understands the proof.”

Mochizuki explained that over many years he had developed a host of tools that he thought would be useful to prove *abc* — but that in the end he realized he did not need all of them.

Some, such as Felipe Voloch of the University of Texas at Austin, were more scathing. “The play showing today at the Hodge Theatre was a farce,” Voloch wrote online, referring to a theoretical construction that Mochizuki named a Hodge Theatre.

Attendees also restated familiar complaints about the proof itself. “The amount of language seems absurd,” said Artur Jackson of Purdue University in West Lafayette, Indiana, at the end of Thursday. And, Voloch told *Nature*: “I don’t know why he chose to make it so abstract.”

A follow-up workshop is expected to take place in Kyoto in July. Kedlaya plans to attend, unlike some of the disillusioned participants in the Oxford workshop. “The claim is an extremely important result,” he says, and the community deserves to know whether it is valid — even though the process will take several more years.

Maths whizz solves a master’s riddle

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