Imagination captured

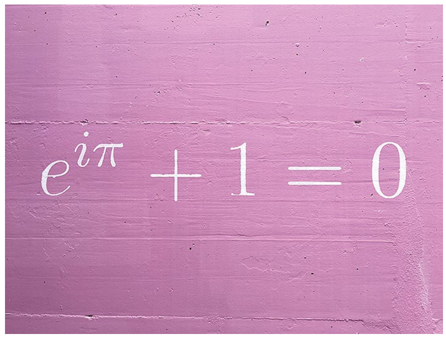
Imaginary numbers have a chequered history, and a sparse — if devoted — following. Abigail Klopper looks at why a concept as beautiful as *i* gets such a bad rap.

When school children delve into the negative abyss that lies beyond zero, few seem to bat an eyelid. But the introduction of imaginary numbers is met largely with fear and confusion by our would-be budding mathematicians. Negative numbers mirror those that can be counted on fingertips, and being down a toy is perhaps a very real prospect for a child — one that is rightly associated with negativity. But what is it about *i* that inspires such mistrust?

There's ample fodder for the argument that it's simply a problem of branding. After all, irrational and radical numbers, having similarly shady connotations, are equally dreaded in early mathematics curricula. But whereas the unfortunate etymology of the irrational and the radical is clearly linked to ratio and root respectively, the naming of *i* has more emotional origins.

Like most good ideas, this one started in Greece, with the mathematician and engineer Heron of Alexandria. Having arrived at the square root of a negative number in one of his calculations, Heron is said to have simply replaced it by its positive value. By the time the Renaissance rolled around, and the ideas of complex calculus had begun to be formalized and written down, the mistrust had already set in. René Descartes was the first to pen the term 'imaginary' in relation to the square root of a negative number, and his peers were similarly derisive. The nomenclature smacks of frustration born of not being able to solve an equation without inventing a new concept. "Just imagine such a thing exists," you can almost hear him cry.

It wasn't until Leonhard Euler showed up that *i* was embraced as something that might be both useful and elegant. And in his wake, a steady stream of admirers followed, as complex analysis was developed and refined into the powerhouse theory it is today. One such admirer was Richard Feynman, who was effusive in his praise for Euler's formula, which casts e^{ix} in terms of trigonometric functions, and in doing so relates algebra to geometry in one fell swoop.



Credit: Laurel Rohde

Equate *x* with π, and the formula reduces to Euler's identity (pictured), which may well be the most compelling proof we have for the existence of a divine being. Indeed, the Internet is full of people proudly sporting tattoos of the centuries-old expression relating everyone's favourite mathematical glyphs: e, *i*, π, 0 and 1.

If Euler is praised for making imaginary numbers elegant, Carl Friedrich Gauss goes down in history as the man who made them accessible. Though not the first to express them graphically, Gauss is responsible for devising the standard notation a + b*i* and, at the tender age of 22, proving the fundamental theorem of algebra.

That the imaginary number completes this theorem is surely one of its more beautiful qualities, despite being precisely the thing that infuriated Descartes the most. Without *i*, there would be no solution to the equation $x^2 + 1 = 0$ and myriad expressions like it. It seems that numbers in the complex plane fail us only in their inability to be ordered. Whereas 4 is undoubtedly greater than 3, it's impossible to compare 5 + 4*i* and 3 – 6*i* in the same way — a small inconvenience to endure for a concept so powerful.

Feynman called Euler's formula an amazing jewel — "one of the most remarkable, almost astounding, formulas in all of mathematics" (R. P. Feynman, *The Feynman Lectures on Physics* Vol. I, Addison-Wesley; 1977). Certainly, the relation lays claim to a unique mixture of the practical and the elegant, allowing physicists to jump between trigonometric and algebraic representations as the mood takes us. Augustin-Louis Cauchy is due similar praise for simplifying the life of the physicist, gifting us the residue theorem, which allows one to integrate by differentiating.

Physics is clearly indebted to complex analysis, and to the very notion of the imaginary. Indeed, quantum mechanics would be difficult to formulate were it not for the complex plane, because the wave function itself is a complex function. So how do we convince the rest of the world — our kids included — that *i* is a number to be revered, not feared?

Maybe the answer is not in rebranding *i*, but in emphasizing the importance of the imaginary in our children's education. After all, where would we be had Descartes not imagined the unimaginable? ❐

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Published online: 1 July 2019
https://doi.org/10.1038/s41567-019-0581-x