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Temperature chaos is present in off-equilibrium spin-glass dynamics

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Experiments featuring non-equilibrium glassy dynamics under temperature changes still await interpretation. There is a widespread feeling that temperature chaos (an extreme sensitivity of the glass to temperature changes) should play a major role but, up to now, this phenomenon has been investigated solely under equilibrium conditions. In fact, the very existence of a chaotic effect in the non-equilibrium dynamics is yet to be established. In this article, we tackle this problem through a large simulation of the 3D Edwards-Anderson model, carried out on the Janus II supercomputer. We find a dynamic effect that closely parallels equilibrium temperature chaos. This dynamic temperature-chaos effect is spatially heterogeneous to a large degree and turns out to be controlled by the spin-glass coherence length ξ. Indeed, an emerging length-scale ξ^* rules the crossover from weak (at $\xi \ll \xi^*$) to strong chaos ($\xi \gg \xi^*$). Extrapolations of ξ^* to relevant experimental conditions are provided.

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An important lesson taught by spin glasses[1] regards the fragility of the glassy phase in response to perturbations such as changes in temperature—temperature chaos (TC)[2–19]—in the couplings[6,7,13,14] or in the external magnetic field[5,20,21]. In particular, it is somewhat controversial[22–27] whether or not TC is the physical mechanism underlying the spectacular rejuvenation and memory effects found in spin glasses[28–31] and several other materials[32–36]. Indeed, a major obstacle in the analysis of these non-equilibrium experiments is that TC is a theoretical notion which is solely defined in an equilibrium context.

Specifically, TC means that the spin configurations that are typical from the Boltzmann weight at temperature T_1 are very atypical at temperature T_2 (no matter how close the two temperatures T_1 and T_2 are).

This equilibrum notion of TC has turned out to be remarkably elusive, even in the context of Mean-Field models (i.e., models that can be solved exactly in the Mean-Field approximation). Indeed, establishing the existence of TC for the Sherrington-Kirkpatrick model has been a real tour de force[12]. Although Sherrington-Kirkpatrick's model is the Mean-Field model of more direct relevance for this work, let us recall for completeness that TC has been investigated as well in other Mean-Field systems named p-spin models. In these models, groups of $p \geq 3$ spins interact (instead, $p = 2$ for Sherrington-Kirkpatrick). Surprisingly enough, one finds different behaviors. On the one hand, we have a recent mathematical proof of the absence of TC in the homogeneous spherical p-spin model[37], in agreement with a previous claim based on physical arguments[38]. On the other hand, TC should be expected when one mixes several values of p[39], as confirmed by a quite recent mathematical analysis[40–43]. Unfortunately, the mathematically rigorous analysis of TC in off-equilibrium dynamics seems out of reach for now, even in the Mean-Field context.

In order to obtain experimentally relevant results, one needs to go beyond the Mean-Field approximation and study short-range spin glasses, represented by the Edwards-Anderson model[44,45]. In this case, analytical investigations are even more difficult, but the equilibrium notion of TC that we have outlined above has been studied through numerical simulations. Yet, these equilibrium simulations have been limited to small system sizes by the severe dynamic slowing down[6–8,11,13,14,16–19].

Here we tackle the problem from a different approach by showing that a non-equilibrium TC effect is indeed present in the dynamics of a large spin-glass sample in three spatial dimensions (our simulations of the Edwards-Anderson model are carried out on the Janus II custom-built supercomputer[46]). In a reincarnation of the statics-dynamics equivalence principle[47–50], just as equilibrium TC is ruled by the system size, dynamic TC is found to be governed by the time-growing spin-glass coherence length $\xi(t_w)$, where the waiting time t_w is the time elapsed since the system was suddenly quenched from some very high temperature to the working temperature T. Below the critical temperature, $T < T_c$, the spin glass is perennially out of equilibrium as evinced by the never-ending (and sluggish) growth of glassy magnetic domains of size $\xi(t_w)$, see refs. [51,52] for instance. Now, the extreme sample-to-sample variations found in small equilibrated systems[16,17,19,53–55] translate into a strong spatial heterogeneity of dynamic TC. Despite such strong fluctuations, our large-scale simulations allow us to observe traces of the effect even in averages over the whole system. In close analogy with equilibrium studies[16], however, dynamic TC can only be fully understood through a statistical analysis of the spatial heterogeneity. A crossover length ξ^* emerges such that TC becomes sizeable only when $\xi(t_w) > \xi^*$. We find that ξ^* diverges when the two observation temperatures T_1 and T_2 approach. The analysis of this divergence reveals that ξ^* is the non-equilibrium partner of the equilibrium chaotic length[3,56]. The large values of $\xi(t_w)$ that we reach with Janus II allow us to perform mild extrapolations to reach the most recent experimental regime[57].

In equilibrium, sample-averaged signals of TC become more visible when the size of the system increases[16]. Analogously, off-equilibrium a weak chaotic effect grows with $\xi(t_w)$ when the whole system is considered on average. Hoping that studying spatial heterogeneities will help us to unveil dynamic TC, we shall consider spatial regions of spherical shape and linear size $\sim \xi(t_w)$, chosen randomly within a very large spin glass. Statics-dynamics equivalence suggests regarding these spheres as the non-equilibrium analog of small equilibrated samples of linear size $\sim \xi(t_w)$. The analogy with equilibrium studies[16,17,19] suggests that a small fraction of our spheres will display strong TC. The important question will be how this rare-event phenomenon evolves as $\xi(t_w)$ grows (in equilibrium, the fraction of samples not displaying TC is expected to diminish exponentially with the number of spins contained in the sample[12,15]).

Results and discussion

Model. We simulate the standard Edwards-Anderson model in a three-dimensional cubic lattice of linear size $L = 160$ and periodic boundary conditions. In each lattice node $\boldsymbol{x}$, we place an Ising spin ($S_{\boldsymbol{x}} = \pm 1$). Lattice nearest-neighbors spins interact through the Hamiltonian $H = -\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} J_{\boldsymbol{xy}} S_{\boldsymbol{x}} S_{\boldsymbol{y}}$. The couplings $J_{\boldsymbol{xy}}$ are independent and identically distributed random variables ($J_{\boldsymbol{xy}} = \pm 1$ with 1/2 probability), fixed when the simulation starts (quenched disorder). This model exhibits a spin-glass transition at temperature $T_c = 1.1019(29)$[58]. We refer to each realization of the couplings as a sample. Statistically independent simulations of a given sample are named replicas. We have considerably extended the simulation of Baity et al.[51], by simulating $N_{\text{Rep}} = 512$ replicas (rather than 256) of the same $N_{\text{S}} = 16$ samples considered in[51], in the temperature range $0.625 \leq T \leq 1.1$.

We simulate the non-equilibrium dynamics with a Metropolis algorithm. In this way, one picosecond of physical time roughly corresponds to a full-lattice Metropolis sweep. At the initial time $t_w = 0$ the spin configuration is fully random (i.e., we quench from infinite temperature). The subsequent growth of spin-glass domains is characterized by the spin-glass coherence length $\xi(t_w)$. Specifically, we use the $\xi_{1,2}$ integral estimators, see refs. [48,51,59,60] for details [the main steps in the computation of $\xi_{1,2}$ are also sketched in Eqs. (8–10)), where one should set $T_1 = T_2 = T$].

Finally, let us briefly comment on our choices for N_{Rep} and N_{S}. A detailed analysis[51,61] shows that, for a given total numerical effort $N_{\text{S}} \times N_{\text{Rep}}$, errors in ξ are minimized if $N_{\text{Rep}} \gg N_{\text{S}}$. Furthermore, Supplementary Note 1 shows that having $N_{\text{Rep}} \gg 1$ is crucial as well for the main quantities considered in this work (see definitions below). Therefore, given our finite computational resources, we have chosen to limit ourselves to $N_{\text{S}} = 16$. This small number of samples is partly compensated by the fact that we are working close to the experimental regime $L \gg \xi$ [we remark that $N_{\text{S}} = 1$ in typical experiments: indeed, statics-dynamics equivalence suggests that the number of statistically independent events is proportional to $N_{\text{S}}(L/\xi)^3$].

The local chaotic parameter. We shall compare the spin textures from temperature T_1 and waiting time t_{w1} with those from temperature T_2 and waiting time t_{w2} (we consider $T_1 \leq T_2 \leq T_c$). A fair comparison requires that the two configurations be ordered at the same lengthscale, which we ensure by imposing the condition

$$\xi(t_{w1}, T_1) = \xi(t_{w2}, T_2) = \xi \ . \qquad (1)$$

A first investigation of TC is shown in Fig. 1. The overlap, computed over the whole sample, of two systems satisfying condition Eq. (1) is used to search for a coarse-grained chaotic effect. The resulting signal is measurable but weak. Instead, as explained in the introduction, spin configurations should be compared locally. Specifically, we consider spherical regions. We start by choosing $N_{\rm sph} = 8000$ centers for the spheres on each

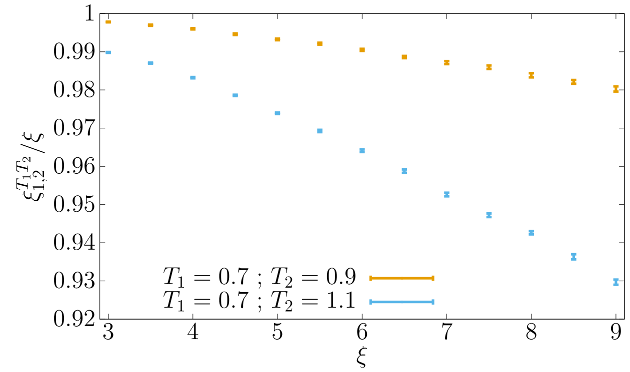

Fig. 1 Non-equilibrium temperature chaos is weak when averaging over the whole system. We compare typical spin configurations at temperature T_1 and time $t_{\rm w1}$ with configurations at T_2 and time $t_{\rm w2}$. The comparison is carried through a global estimator of the coherence length of their overlap $\xi_{1,2}^{T_1T_2}$ which represents the maximum lengthscale at which configurations at temperatures T_1 and T_2 still look similar, see Methods section for further details. The two times $t_{\rm w1}$ and $t_{\rm w2}$ are chosen in such a way that the configurations at both temperatures have glassy-domains of the same size, namely $\xi_{1,2}(t_{\rm w1}, T_1) = \xi_{1,2}(t_{\rm w2}, T_2) = \xi$. The figure shows the ratio $\xi_{1,2}^{T_1T_2}/\xi$ as a function of ξ for two pairs of temperatures (T_1, T_2), recall that $T_{\rm c} \approx 1.1$[58]. Under the hypothesis of fully developed Temperature Chaos, we would expect $\xi_{1,2}^{T_1T_2}$ to be negligible compared to ξ. Instead, our data shows only a small decrease of $\xi_{1,2}^{T_1T_2}/\xi$ with growing ξ (the larger the difference $T_2 - T_1$ the more pronounced the decrease). Error bars represent one standard deviation.

sample. The spheres' centers are chosen randomly, with uniform probability, on the dual lattice which, in a cubic lattice with periodic boundary conditions, is another cubic lattice of the same size, also periodic boundary condition. The nodes of the dual lattice are the centers of the elementary cells of the original lattice. The radii of the spheres are varied, but their centers are held fixed. Let $B_{s,r}$ be the s-th ball of radius r. Our basic observable is the overlap between replica σ (at temperature T_1), and replica $\tau \neq \sigma$ (at temperature T_2):

$$q_{T_1,T_2}^{s,r;\sigma,\tau}(\xi) = \frac{1}{N_r} \sum_{\boldsymbol{x} \in B_{s,r}} s_{\boldsymbol{x}}^{\sigma,T_1}(t_{\rm w1}) s_{\boldsymbol{x}}^{\tau,T_2}(t_{\rm w2}), \qquad (2)$$

where N_r is the number of spins in the ball, and $t_{\rm w1}$ and $t_{\rm w2}$ are chosen according to Eq. (1). Averages over thermal histories, indicated by $\langle \ldots \rangle_T$, are computed by averaging over σ and τ.

Next, we generalize the so-called chaotic parameter[6,16,17,20] as

$$X_{T_1,T_2}^{s,r}(\xi) = \frac{\langle [q_{T_1,T_2}^{s,r;\sigma,\tau}(\xi)]^2 \rangle_T}{\sqrt{\langle [q_{T_1,T_1}^{s,r;\sigma,\tau}(\xi)]^2 \rangle_T \, \langle [q_{T_2,T_2}^{s,r;\sigma,\tau}(\xi)]^2 \rangle_T}}, \qquad (3)$$

The extremal values of the chaotic parameter have a simple interpretation: $X_{T_1,T_2}^{s,r} = 1$ corresponds with a situation in which spin configurations in the ball $B_{s,r}$, at temperatures T_1 and T_2, are completely indistinguishable (absence of chaos) while $X_{T_1,T_2}^{s,r} = 0$ corresponds to completely different configurations (strong TC). A representative example our results is shown in Fig. 2.

Our main focus will be on the distribution function $F(X, T_1, T_2, \xi, r) =$ Probability $[X_{T_1,T_2}^{s,r}(\xi) < X]$ and on its inverse $X(F, T_1, T_2, \xi, r)$.

The rare-event analysis. Representative examples of distribution functions $F(X, T_1, T_2, \xi, r)$ are shown in Fig. 3. We see that, in close analogy with equilibrium systems[16,17,19], while most spheres exhibit a very weak TC ($X > 0.9$, say), there is a fraction of spheres

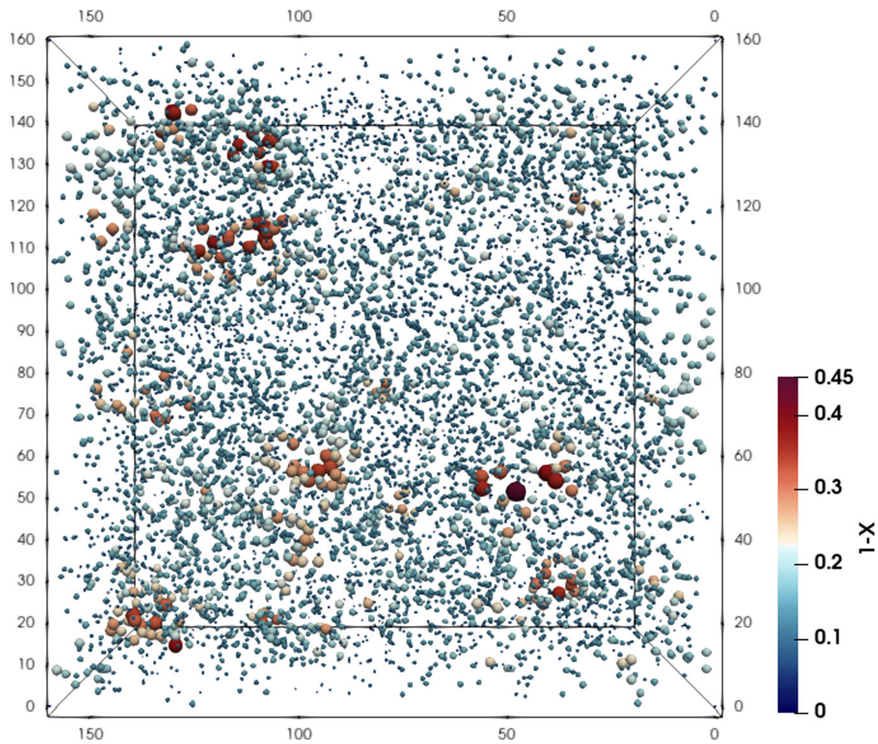

Fig. 2 Dynamic temperature chaos is spatially heterogeneous. The 8000 randomly chosen spheres in a sample of size $L = 160$ are depicted with a color code depending on $1 - X$ [X is the chaotic parameter, Eq. (3), as computed for spheres of radius $r = 12$, $\xi = 12$ and temperatures $T_1 = 0.7$ and $T_2 = 1.0$]. For visualization purposes, spheres are represented with a radius $12(1 - X)$, so that only fully chaotic spheres (i.e., $X = 0$) have their real size.

displaying smaller X (stronger chaos). Note that the probability F of finding spheres with X smaller than any prefixed value increases when ξ grows.

In order to make the above finding quantitative, we consider the (inverse) distribution function $X(F, T_1, T_2, \xi, r)$. We start by fixing (T_1, T_2), ξ and some small probability F, which leaves us with a function of only r. In order to obtain smoother interpolations for small radius, however, we have used $N_r^{1/3}$ instead of r as our independent variable, a technical detailed discussion can be found in Supplementary Note 3.

Figure 4 shows plots of $1 - X$ under these conditions, which exhibit well-defined peaks (see further information about the fitting function to the peaks in the Supplementary Note 2). Now, to a first approximation we can characterize any peak by its position, height and width. Fortunately, these three parameters turn out to describe the scaling with ξ of the full $1 - X$ curve, see Supplementary Note 4.

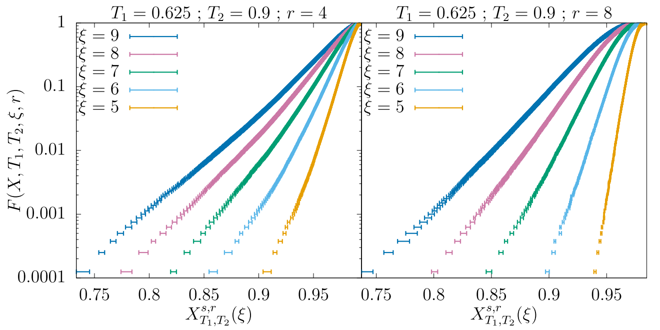

Fig. 3 Temperature chaos increases with the coherence length. The figure shows the distribution function $F(X, T_1, T_2, \xi, r)$ for temperatures $T_1 = 0.625$ and $T_2 = 0.9$, for spheres of radius $r = 4$ and $r = 8$, as computed for various values of ξ. The distributions have been extrapolated to infinite number of replicas $N_{\text{Rep}} = \infty$, see Supplementary Note 1 for further details. Error bars, that represent one standard deviation, are horizontal, because we have actually extrapolated the chaotic parameter, which is its inverse function $X(F, T_1, T_2, \xi, r)$. Most of the spheres have a chaotic parameter very close to $X = 1$ (absence of chaos). However, if we fix our attention, for instance, on percentile 1 (i.e., $F = 0.01$) we see that the corresponding value of X decreases monotonically (and significantly) as ξ grows, signaling a developing chaotic effect. This trend is clear both for spheres of radius $r = 4$ and $r = 8$.

The physical interpretation of the peak's parameters is clear. The peak's height represents the strength of dynamic TC (the taller the peak, the larger the chaos). The peak's position indicates the optimal lengthscale for the study of TC, given the probability F, ξ and the temperatures T_1, T_2. The peak's width indicates how critical it is to spot this optimal lengthscale (the wider the peak, the less critical the choice). Perhaps unsurprisingly, the peak's position is found to scale linearly with ξ, while the peak's width scales as ξ^β, with β slightly larger than one, see Supplementary Note 5 for further details. We shall focus here on the temperature and ξ dependence of the peak's height (i.e., the strength of chaos), which has a richer behavior.

The ξ dependence of the peak's height (for a given probability F and temperatures T_1 and T_2) turns out to be reasonably well described by the following ansatz:

$$f_{\max}(\xi) = \frac{\varepsilon(\xi)}{1 + \varepsilon(\xi)} \text{ , with } \varepsilon(\xi) = (\xi/\xi^*)^\alpha \text{ .} \tag{4}$$

This formula describes a crossover phenomenon, ruled by a characteristic length ξ^*. For $\xi \ll \xi^*$ the peak's height grows with ξ as a power law, while for $\xi \gg \xi^*$ the strong-chaos limit [i.e., $(1 - X) \to 1$] is approached. However, some consistency requirements should be met before taking the crossover length ξ^* seriously. Not only should the fit to Eq. (4) be of acceptable statistical quality (the fit parameters are the characteristic lengthscale ξ^* and the exponent α). One would also wish exponent α to be independent of the temperatures T_1 and T_2 and of the chosen probability F.

We find fair fits to Eq. (4), see Table 1. In all cases, exponent α turns out to be compatible with 2.1 at the two-σ level [except for the $(F = 0.01, T_1 = 0.625, T_2 = 0.8)$ fit]. Under these conditions, we can interpret ξ^* as a characteristic length indicating the crossover from weak to strong TC, at the probability level indicated by F. Furthermore, the relatively large value of exponent α indicates that this crossover is sharp.

The trends for the crossover length ξ^* in Table 1 are very clear: ξ^* grows upon increasing F or upon decreasing $T_2 - T_1$. Identifying ξ^* as the non-equilibrium partner of the equilibrium chaotic length $\ell_c(T_1, T_2)$[3,56] will allow us to be more quantitative (indeed, the two lengthscales indicate the crossover between weak

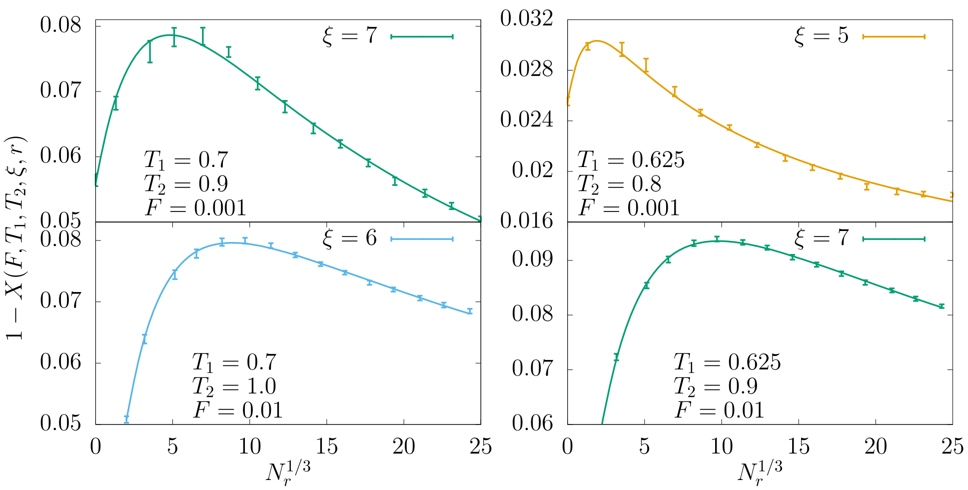

Fig. 4 Emergence of an optimal scale to observe temperature chaos. The difference $1 - X(F, T_1, T_2, \xi, r)$ [recall that $X(F, T_1, T_2, \xi, r)$ is the inverse of the distribution function] as a function of the cubic root $N_r^{1/3}$ of the number of spins in the spheres, as computed for different values of the probability level F, the temperatures T_1 and T_2, and the coherence length ξ. In this representation, the optimal size of the spheres for the observation of chaos (for given parameters F, T_1, T_2 and ξ) appears as the maximum of the curves. Continuous lines are fits to a smooth interpolating function, further details can be found in Supplementary Note 2. Error bars represent one standard deviation.

Table 1 Parameters describing the crossover between weak and strong temperature chaos regimes.

F	T_1	T_2	ξ_{min}	ξ^*	α	χ^2/d.o.f
0.001	0.625	0.7	4.75	55(4)	2.10(7)	14.10/19
0.001	0.625	0.8	5.25	24.1(8)	2.18(6)	22.67/17
0.001	0.625	0.9	4.75	16.8(3)	2.09(4)	28.88/19
0.001	0.625	1.0	4.75	13.24(15)	2.04(3)	8.77/19
0.001	0.7	0.8	4.75	43.5(15)	2.12(5)	41.05/28
0.001	0.7	0.9	4.75	22.9(5)	2.09(4)	33.32/28
0.001	0.7	1.0	4.75	16.3(2)	2.04(4)	22.32/28
0.01	0.625	0.8	5.75	29.3(5)	2.21(3)	13.32/15
0.01	0.625	0.9	5.75	20.5(3)	2.12(2)	16.05/15
0.01	0.625	1.0	4.75	15.87(16)	2.08(2)	23.93/19
0.01	0.7	0.8	4.75	51.4(12)	2.17(3)	8.06/28
0.01	0.7	0.9	5.25	27.9(4)	2.11(2)	31.56/26
0.01	0.7	1.0	4.75	19.9(2)	2.05(2)	31.78/28

Parameters obtained in the fits to Eq. (4) of our data for the peak's height, see Fig. 4, with $\xi_{min} \le \xi \le \xi_{max}$. We also report the fits' figure of merit χ^2/d.o.f. We chose ξ_{min} by requiring a P value greater than 0.05 in the fits ($\xi_{max} = 9.5$ for $T_1 = 0.625$ and $\xi_{max} = 12.5$ for $T_1 = 0.7$). T_1 and T_2 represent the temperatures involved in the computation of the chaotic parameter. Unfortunately, the flatness of the peak for ($T_1 = 0.625$, $T_2 = 0.7$, $F = 0.01$) did not allow us to compute the peak's parameters.

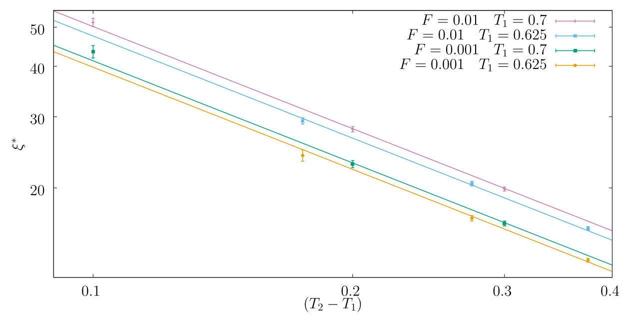

Fig. 5 Universal scaling of dynamic chaos. The characteristic length ξ^* is plotted against the temperature difference $T_2 - T_1$ in a log-log scale. Each curve is uniquely identified by the probability level F and the smallest temperature of each pair T_1. Fits to Eq. (6), enforcing a common exponent, are shown with continuous lines and result in a chaotic exponent $\zeta_{NE} = 1.19(2)$. Error bars represent one standard deviation.

chaos and strong chaos). Now, the equilibrium $\ell_c(T_1, T_2)$ has been found to scale for the 3D Ising spin glass as

$$\ell_c(T_1, T_2) \propto (T_2 - T_1)^{-1/\zeta} \quad , \tag{5}$$

with $\zeta \approx 1.07$[14] or $\zeta \approx 1.07(5)$[16]. These considerations suggest the following ansatz for the non-equilibrium crossover length

$$\xi^*(T_1, T_2, F) = B(F, T_1)\ (T_2 - T_1)^{-1/\zeta_{NE}} \quad , \tag{6}$$

where $B(F, T_1)$ is an amplitude. We have tested Eq. (6) by computing a joint fit for four (T_1, F) pairs as functions of $T_2 - T_1$, allowing each curve to have its own amplitude but enforcing a common ζ_{NE} (see Fig. 5). The resulting χ^2/d.o.f. = 7.55/7 validates our ansatz, with an exponent $\zeta_{NE} = 1.19(2)$ fairly close to the equilibrium result $\zeta = 1.07(5)$[16]. This agreement strongly supports our physical interpretation of the crossover length. We, furthermore, find that B is only weakly dependent on T_1. Nevertheless, the reader should be warned that it has been suggested[16] that the equilibrium exponent ζ may be different in the weak- and strong-chaos regimes.

Conclusions

We have shown that the concept of temperature chaos can be meaningfully extended to the non-equilibrium dynamics of a large spin glass. This is, precisely, the framework for rejuvenation and memory experiments[28–31], as well as other more chaos-oriented experimental work[57]. Therefore, our precise characterization of dynamical temperature chaos paves the way for the interpretation of these and forthcoming experiments. Our simulation of spin-glass dynamics doubles the numerical effort in[51] and has been carried out on the Janus-II special-purpose supercomputer.

The key quantity governing dynamic temperature chaos is the time-dependent spin-glass coherence length $\xi(t_w)$. The very strong spatial heterogeneity of this phenomenon is quantified through a distribution function F. This probability can be thought of as the fraction of the sample that shows a chaotic response to a given degree. When comparing temperatures T_1 and T_2, the degree of chaoticity is governed by a lengthscale $\xi^*(F, T_1, T_2)$. While chaos is very weak if $\xi(t_w) \ll \xi^*(F, T_1, T_2)$, it quickly becomes strong as $\xi(t_w)$ approaches $\xi^*(F, T_1, T_2)$. We find that, when T_1 approaches T_2, $\xi^*(F, T_1, T_2)$ appears to diverge with the same critical exponent that it is found for the equilibrium chaotic length[16].

Although we have considered in this work fairly small values of the chaotic system fraction F, a simple extrapolation, linear in log F, predicts $\xi^* \approx 60$ for $F = 0.1$ at $T_1 = 0.7$ and $T_2 = 0.8$ (our closest pair of temperatures in Table 1). A spin-glass coherence length well above $60a_0$ is experimentally reachable nowadays[52,57,62,63] (a_0 is the typical spacing between spins), which makes our dynamic temperature chaos significant. Indeed, while completing this manuscript, a closely related experimental study[57] reported a value for exponent ζ_{NE} in fairly good agreement with our result of $\zeta_{NE} = 1.19(2)$ in Fig. 5.

Let us conclude by commenting on possible venues for future research. Clearly, it will be important to understand in detail how dynamic temperature chaos manifests itself in non-equilibrium experiments. Simple protocols (in which temperature sharply drops from T_2 to T_1, see, e.g., Zhai et. al.[57]) seems more accessible to a first analysis than memory and rejuvenation experiments[28–31]. An important problem is that the correlation functions that are studied theoretically are not easily probed experimentally. Instead, experimentalists privilege the magnetization density (which is a spatial average over the whole sample). Therefore an important theoretical goal is to predict the behavior of the non-equilibrium time-dependent magnetization upon a temperature drop. One may speculate that the Generalized Fluctuation-Dissipation Relations[64] might be the route connecting the correlation functions with the response to an externally applied magnetic field. Interestingly enough, these relations (that apply at fixed temperature) can be defined locally as well[65]. The resulting spatial distribution function allows the reconstruction of the global response to the magnetic field. Extending this analysis to a temperature drop may turn out to be fruitful in the future.

Methods

All the observables involved in the computation of temperature chaos depend on a pair of replicas (σ, τ). The basic quantity is the overlap field

$$q^{\sigma,\tau}(\boldsymbol{x}, t_w) = s_{\boldsymbol{x}}^{\sigma}(t_w) s_{\boldsymbol{x}}^{\tau}(t_w), \tag{7}$$

Usually, this pair of replicas are at the same temperature T. All the definitions are, however, straightforwardly extended to two temperatures. For instance, the four-point two-temperature spatial correlation function is

$$C_4^{T_1 T_2}(T_1, T_2, t_{w1}, t_{w2}, \boldsymbol{r}) = \left[\langle q^{\sigma(T_1),\tau(T_2)}(\boldsymbol{x}, t_{w1}, t_{w2}) q^{\sigma(T_1),\tau(T_2)}(\boldsymbol{x} + \boldsymbol{r}, t_{w1}, t_{w2}) \rangle_T \right]_J, \tag{8}$$

where $[\ldots]_J$ denotes the average over the samples. Building on this function we can define our integral estimator for the coherence length[60]:

$$I_k^{T_1T_2}(t_{w1}, t_{w2}) = \int_0^\infty r^k \, C_4^{T_1T_2}(r, t_{w1}, t_{w2}) \, dr, \tag{9}$$

and

$$\xi_{k,k+1}^{T_1T_2}(t_{w1}, t_{w2}) = \frac{I_{k+1}^{T_1T_2}(t_{w1}, t_{w2})}{I_k^{T_1T_2}(t_{w1}, t_{w2})}. \tag{10}$$

As explained in the main text, times t_{w1} and t_{w2} are fixed through the condition expressed in Eq. (1), which ensures that we are comparing spin configurations that are ordered on the same length scale.

Since our t_w are on a discrete grid, we solve Eq. (1) for the global overlaps through a (bi)linear interpolation.

Data availability

The data contained in the figures of this paper, accompanied by the gnuplot script files that generate these figures, are publicly available at https://github.com/JanusCollaboration/caosdin.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Code availability

The codes that support the findings of this study are available from the corresponding author upon reasonable request.

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Author contributions

J.M.G.-N. and D.N. contributed to Janus/Janus II simulation software. D.I., A.T. and R.T. contributed to Janus II design. M.B.-J., E.C., A.C., L.A.F, J.M.G.-N., I.G.-A.P., A.G.-G., D.I., A.M., A.M.-S., I.P., S.P.-G., S.F.S., A.T. and R.T. contributed to Janus II hardware and software development. L.A.F., V.M.-M. and J.M.-G. designed the research. J.M.-G. analyzed the data. M.B.-J., L.A.F., E.M., V.M.-M., J.M.-G., I.P., G.P., B.S., J.J.R.-L., F.R.-T. and D.Y. discussed the results. V.M.-M., J.M.-G., B.S. and D.Y. wrote the paper.

Competing interests

The authors declare no competing interests.

Additional information

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